The Elusive Perfect Problem

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Good and Ba Problems

Example: Trapezoida Numbers

Example: Codes and Communication

"Do No Harm" activities in an enrichment program for "unenriched" students

Paul Zeitz

University of San Francisco San Francisco Math Circle

Jun. 3, 2011

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Started Fall 2005

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Started Fall 2005

 Matthias Beck (SFSU), Brandy Wiegers (MSRI), PZ (USF)

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MSRI

Generous Donors

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- Matthias Beck (SFSU), Brandy Wiegers (MSRI), PZ (USF)
- MSRI
- Generous Donors
- Instructors, Teachers, Students, Parents

What Type of Circle?

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Example: Trapezoida Numbers

Example: Codes and Communication Students: Unenriched —— Already enriched Diversity: High —— Low Recruitment: Teacher — Self — Parents Time: After-school — Evening/Weekend Length: Short —— Long Level: Math doesn't suck! —— Olympiad Instruction: Small groups —— Pure Lecture

What Type of Circle?

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Example: Trapezoida Numbers

Example: Codes and Communication Students: Unenriched — Already enriched
Diversity: High — Low
Recruitment: Teacher — Self — Parents
Time: After-school — Evening/Weekend
Length: Short (50 min) — Long
Level: Math doesn't suck! — Olympiad

Instruction: Small groups —— Pure Lecture

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Anything that inhibits

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Anything that inhibits Confidence

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Anything that inhibits

- Confidence
- Conversation/Argument
- Quick mathematical feedback

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Anything that inhibits

- Confidence
- Conversation/Argument
- Quick mathematical feedback
- Physical, tangible interaction with the world

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Anything that inhibits

- Confidence
- Conversation/Argument
- Quick mathematical feedback
- Physical, tangible interaction with the worldINVESTIGATION

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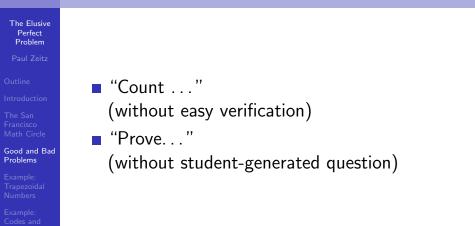
Example: Trapezoidal Numbers

Example: Codes and Communication ■ "Count . . . "

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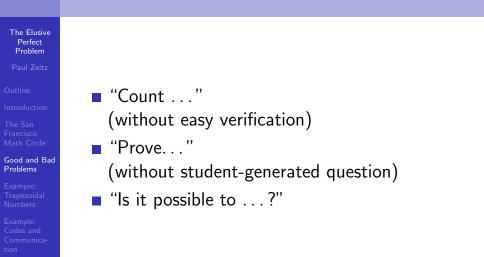
Examples of Bad Problems The Elusive Perfect Problem "Count ..." (without easy verification) Good and Bad Problems

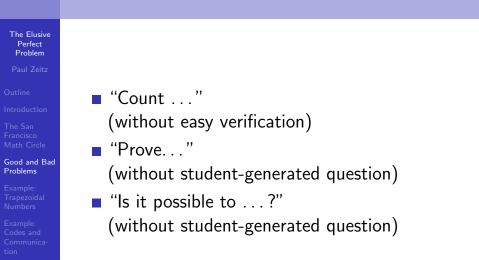
Examples of Bad Problems The Elusive Perfect Problem "Count" (without easy verification) "Prove..." Good and Bad Problems



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Two Styles for Good Problems The Elusive Perfect Problem Hard, but with "scaffolding" Good and Bad Problems

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■ Hard, but with "scaffolding"

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Warm-up problems

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■ Hard, but with "scaffolding"

- Warm-up problems
- Hint rationing

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■ Hard, but with "scaffolding"

- Warm-up problems
- Hint rationing
- Trained helpers

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Hard, but with "scaffolding"

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- Warm-up problems
- Hint rationing
- Trained helpers

Easier, stand-alone

Example:	Trapezoidal	Numbers
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Example: Trapezoidal Numbers

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Example: Trapezoidal Numbers

Example: Codes and Communication A number is *trapezoidal* if it can be expressed as a sum of consecutive positive integers.

Example: Trapezoidal Numbers

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Example: Trapezoidal Numbers

Example: Codes and Communication A number is *trapezoidal* if it can be expressed as a sum of consecutive positive integers.

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Find all trapezoidal numbers.

Example: Trapezoidal Numbers

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Example: Trapezoidal Numbers

Example: Codes and Communication A number is *trapezoidal* if it can be expressed as a sum of consecutive positive integers.

- Find all trapezoidal numbers.
- Answer: all positive integers, except 1, 2, 4, 8, 16,

Example: Trapezoidal Numbers

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Example: Trapezoidal Numbers

Example: Codes and Communication A number is *trapezoidal* if it can be expressed as a sum of consecutive positive integers.

- Find all trapezoidal numbers.
- Answer: all positive integers, except 1, 2, 4, 8, 16,
- Bad: Algebra (my plan)

Example: Trapezoidal Numbers

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Example: Trapezoidal Numbers

Example: Codes and Communication

- A number is *trapezoidal* if it can be expressed as a sum of consecutive positive integers.
- Find all trapezoidal numbers.
- Answer: all positive integers, except 1, 2, 4, 8, 16,
- Bad: Algebra (my plan)
- Good: What the students invented (dots)

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Example:
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Example: Trapezoidal Numbers

Example: Codes and Communication $T = \frac{(a+\ell)}{2}n$

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Example: Trapezoidal Numbers

Example: Codes and Communication

$$T = \frac{(a+\ell)}{2}n$$
$$T = \frac{(2a+n-1)}{2}n$$



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Good and Ba Problems

Example: Trapezoidal Numbers

Example: Codes and Communication $T = \frac{(a+\ell)}{2}n$ $T = \frac{(2a+n-1)}{2}n$ $T = \frac{odd \cdot even}{2}$

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Example: Trapezoidal Numbers

Example: Codes and Communication • $T = \frac{(a + \ell)}{2}n$ • $T = \frac{(2a + n - 1)}{2}n$ • $T = \frac{odd \cdot even}{2}$ • The smaller of these two factors equals *n*; the larger equals $a + \ell$.

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Example: Trapezoidal Numbers

Example: Codes and Communication ■ *T* = 36

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Example: Trapezoidal Numbers

Example: Codes and Communication **T** = 36 $2T = 72 = 8 \cdot 9$

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Example: Trapezoidal Numbers

Example: Codes and Communication ■ *T* = 36

2T = 72 = 8 ⋅ 9
n = 8, a + ℓ = 9

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Example: Trapezoidal Numbers

Example: Codes and Communication ■ *T* = 36

 $\bullet 2T = 72 = 8 \cdot 9$

■ *n* = 8, *a* + *l* = 9

• T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8.

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Example: Trapezoidal Numbers

Example: Codes and Communication ■ *T* = 36

- $2T = 72 = 8 \cdot 9$
- *n* = 8, *a* + *l* = 9
- T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8.

■ *T* = 22

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Example: Codes and Communication ■ *T* = 36

■ $2T = 72 = 8 \cdot 9$ ■ $n = 8, a + \ell = 9$

• T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8.

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2 $T = 44 = 4 \cdot 11$

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Example: Trapezoidal Numbers

■ *T* = 36

■
$$2T = 72 = 8 \cdot 9$$

■ $n = 8, a + \ell = 9$

•
$$T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$$

•
$$2T = 44 = 4 \cdot 11$$

• $n = 4, a + \ell = 11$

■
$$n = 4, a + \ell = 1$$

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■ $2T = 72 = 8 \cdot 9$ ■ $n = 8, a + \ell = 9$

T = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8.

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■ *T* = 22

■ $2T = 44 = 4 \cdot 11$

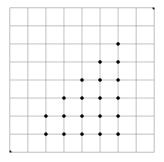
 $\bullet \quad n=4, a+\ell=11$

• T = 4 + 5 + 6 + 7.

Dots to the Rescue! 2 + 3 + 4 + 5 + 6 = ?



Example: Codes and Communication



$2 + 3 + 4 + 5 + 6 = 5 \cdot 4$



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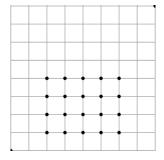
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Example: Codes and Communication





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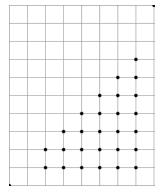
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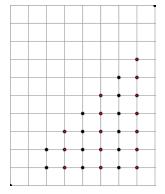
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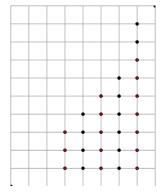
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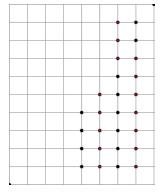
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$2 + 3 + 4 + 5 + 6 + 7 = 3 \cdot 9$



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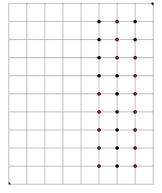
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	$13 \times 4 = 1$?									
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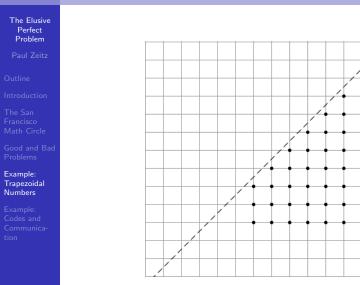
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	13 × 4 =?
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	13 × 4 =?
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$13 \times 4 = 3 + 4 + \dots + 9 + 10$



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Warmup problem I: "Do you know what I know?" (Zvonkin)

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Warmup problem I: "Do you know what I know?" (Zvonkin)

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■ Opaque cards are labeled 1/2, 2/3, or 3/4.

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Example: Codes and Communication Warmup problem I: "Do you know what I know?" (Zvonkin)

- Opaque cards are labeled 1/2, 2/3, or 3/4.
- Two opposing players sit opposite one another

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Example: Codes and Communication Warmup problem I: "Do you know what I know?" (Zvonkin)

- Opaque cards are labeled 1/2, 2/3, or 3/4.
- Two opposing players sit opposite one another
- Moderator holds up a card so that each player sees one side.

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Example: Trapezoidal Numbers

Example: Codes and Communication Warmup problem I: "Do you know what I know?" (Zvonkin)

- Opaque cards are labeled 1/2, 2/3, or 3/4.
- Two opposing players sit opposite one another
- Moderator holds up a card so that each player sees one side.
- First player to say which number her opponent sees wins.

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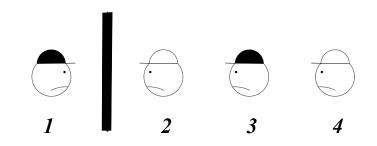
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■ Warmup problem II : Heads in the Sand



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	First	"hard"	problem		
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	First "hard" problem
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Example: Codes and Communication Ten people are lined up, all facing forward.
Hats are placed on them (black or white, no pattern).

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Example: Codes and Communication Ten people are lined up, all facing forward.

- Hats are placed on them (black or white, no pattern).
- A person can ONLY see the hat colors of the people in front of him or her.

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Example: Codes and Communication

- Ten people are lined up, all facing forward.
- Hats are placed on them (black or white, no pattern).
- A person can ONLY see the hat colors of the people in front of him or her.
- Starting from the rear, each person will say what color their hat is. The moderator will tell them if they are right or wrong. They are ONLY allowed to say "black" or "white."

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Example: Codes and Communication Assume that they can meet for a strategy meeting before the hats are put on. How can they maximize the number of correct answers?

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	First	''hard''	problem		
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	First "hard" problem
The Elusive Perfect Problem Paul Zeitz	
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The San Francisco Math Circle	Crux idea is parity. How to hint this?
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First "hard" problem The Elusive Perfect Problem Crux idea is parity. How to hint this? With **MORE PROBLEMS!** Example: Codes and

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Communica-

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Example: Codes and Communication

Crux idea is parity. How to hint this? With MORE PROBLEMS!

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Trained student helpers/performers